LET'S CONNECT PROBABILITIES TO GRAPHS!

Erik-Jan

December 7, 2017

Causality Reading Group

CONTENTS

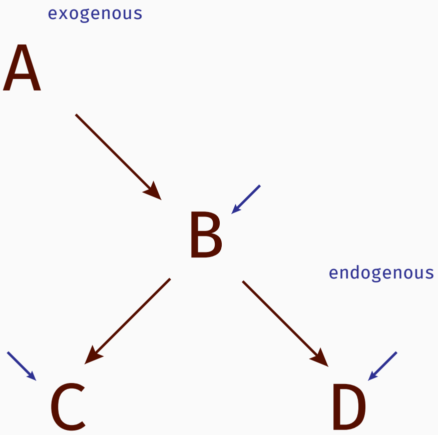
Determinism

Axioms

- Causal Markov Condition
- Causal Minimality Condition
- Faithfulness Condition

DETERMINISM

DETERMINISM



Three levels of determinism in a causal structure:

- Deterministic - endogenous variables fully defined by their parents (and thus, by the exogenous variables)
- Indeterministic - endogenous variables have random noise
- Pseudoindeterministic - There exists a graph over a superset of the variables that is deterministic but we don't know it.

"Structural equation models in the social sciences are usually assumed to be pseudoindeterministic causal structures." (p. 28)

AXIOMS

CAUSAL MARKOV CONDITION

Let **V** be a set of variables.
Let **G** be a causal graph over **V**.
Let **P** be a probability distribution over **V**.

G and **P** satisfy the Causal Markov Condition iff for every $W \in \mathbf{V}$,

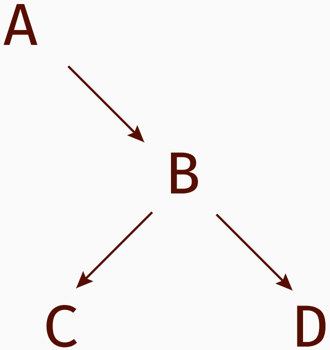
$$W \perp\!\!\!\perp \mathbf{V} \setminus \text{Descendants}(W) \mid \text{Parents}(W)$$

CAUSAL MARKOV CONDITION

$C \perp\!\!\!\perp \{A, D\} | B$

$D \perp\!\!\!\perp \{A, C\} | B$

$B \perp\!\!\!\perp \emptyset | A$



CAUSAL MARKOV CONDITION

If the Causal Markov Condition holds, then we can factorise $\mathbf{P}$:

$$P(\mathbf{V}) = \prod_{W \in \mathbf{V}} P(W|\text{Parents}(W))$$

so in our example:

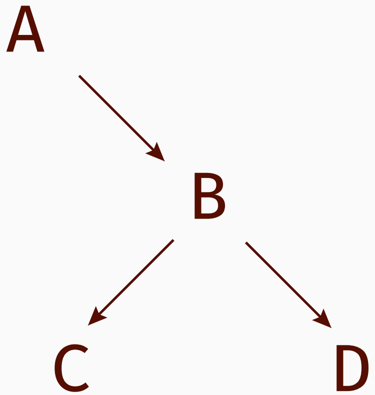
$$P(\mathbf{V}) = P(A)P(B|A)P(C|B)P(D|B)$$

neat!

The grand axiom of:

"Don't add arrows which are not supposed to be there"

Formally: for a given $P(V)$, if any subgraph of G over all of V exists for which the Causal Markov Condition holds, then G is not causally minimal.


A
B
C
D

The big one:

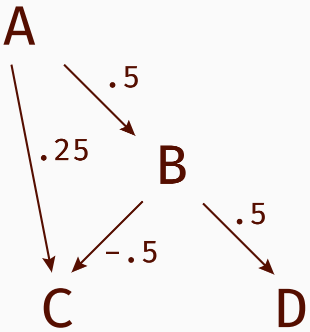
> ”P and G are faithful to one another iff all independences in P(V) are implied by the markov condition applied to G.”
>
> ”P(V) is faithful if a DAG G exists to which it is faithful.”

”Important to discovering causal structure”.

Can we break faithfulness?

A wild question appears!

Will a multivariate normal probability distribution generated from this linear pseudoindeterministic causal structure be faithful to it?



I tested using R:

```r
mod <- "
B ~ 0.5*A
C ~ -0.5*B + 0.25*A
D ~ 0.5*B
"
d <- lavaan::simulateData(mod, sample.nobs = 1e7L)
cor(d$A, d$C)

# [1] 0.000120153
```

Figure: Sorry for all the pixels, LaTeX was not cooperating...