Extending SEM using insights from deep learning

Erik-Jan van Kesteren & Daniel Oberski

Utrecht University, Methodology & Statistics

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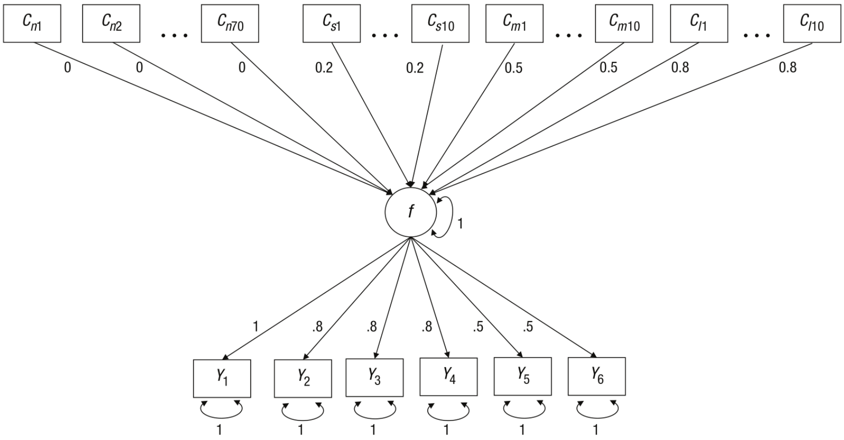
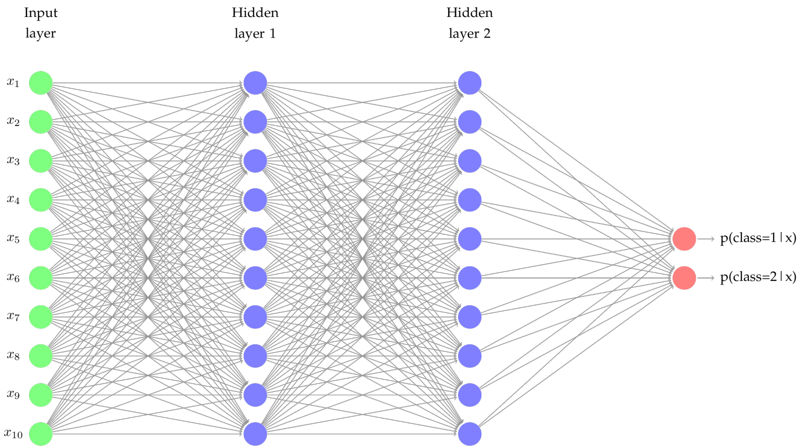


Jacobucci, Brandmaier & Kievit (2019)



http://mdtux89.github.io/2015/12/11/torch-tutorial.html (2015)

- Some SEM models are overparameterized (e.g., when $p > n$)
- We can't estimate these models with default SEM
- Neural networks can be extremely overparameterized
- Deep learning software (e.g., TensorFlow) can still estimate these
- Can we use deep learning methods for SEM?

Computation graphs

The SEM computation graph

Extending SEM

R package showcase

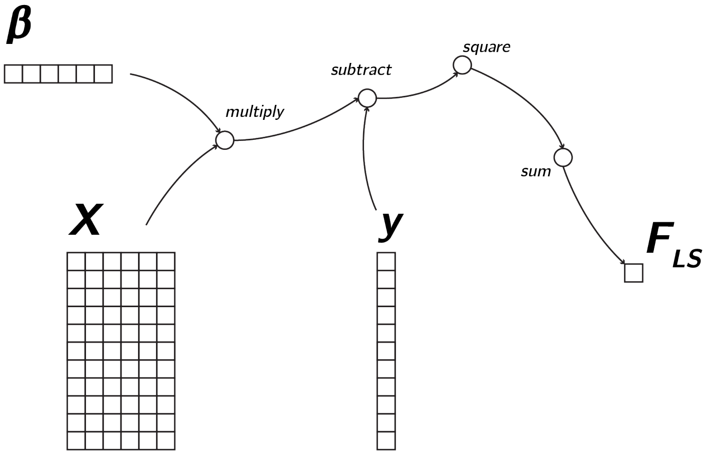
Computation graphs

Computation graphs

Describe operations from parameters to loss function

$$\boldsymbol{\theta} \to F(\boldsymbol{\theta})$$

Least squares regression computation graph



Computation graphs

Software can automatically compute $\nabla F(\boldsymbol{\theta})$ (autograd)
Software implements optimisation algorithms (e.g., Adam)

Computation graphs

Computation graph $+$ software $\rightarrow$ parameter estimation

The SEM computation graph

SEM computation graph

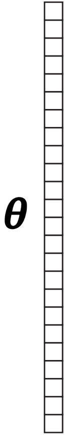
Describe operations from parameters to loss function

$$\boldsymbol{\theta} \to F(\boldsymbol{\theta})$$

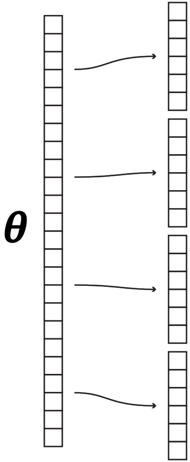
SEM computation graph

$$
\begin{aligned}
\boldsymbol{\theta} &= \{B_{\mathbf{0}}, \mathbf{\Lambda}, \mathbf{\Psi}, \mathbf{\Theta}\} \\
B &= (I - B_{\mathbf{0}}) \\
\mathbf{\Sigma} &= \mathbf{\Lambda} B^{-1} \mathbf{\Psi} B^{-T} \mathbf{\Lambda}^{T} + \mathbf{\Theta} \\
F_{ML}(\boldsymbol{\theta}) &= log|\mathbf{\Sigma}| + tr\left[S\mathbf{\Sigma}^{-1}\right]
\end{aligned}
$$

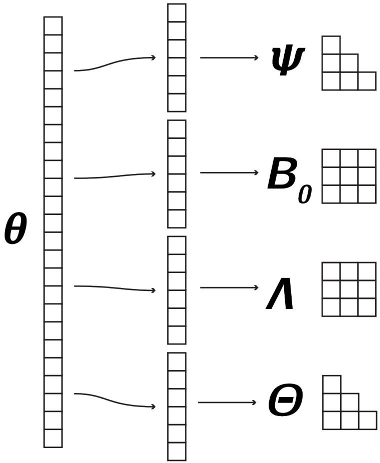
SEM computation graph



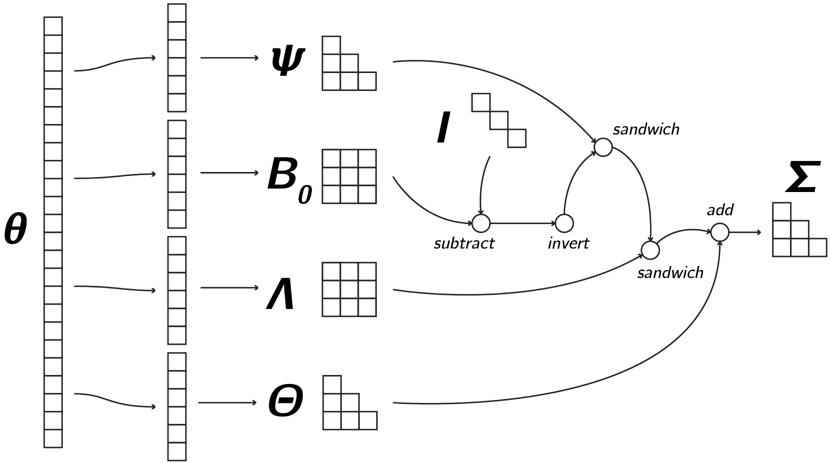
SEM computation graph



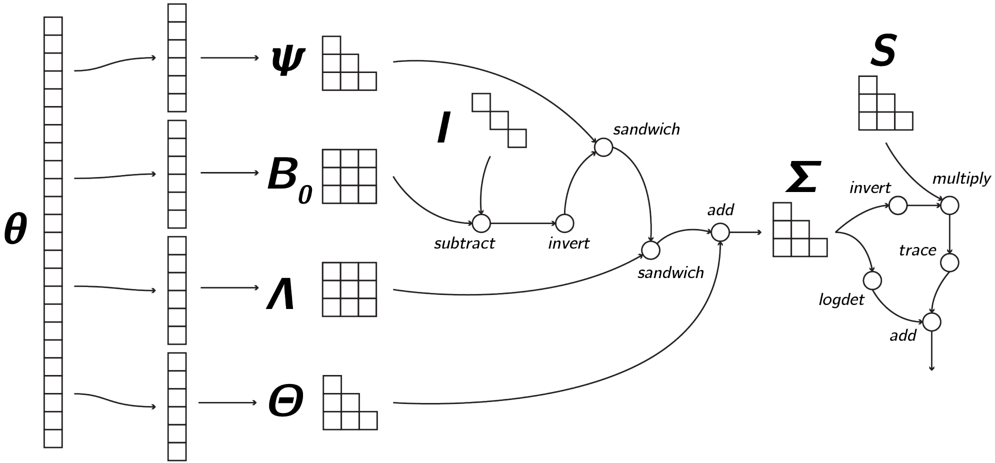
SEM computation graph



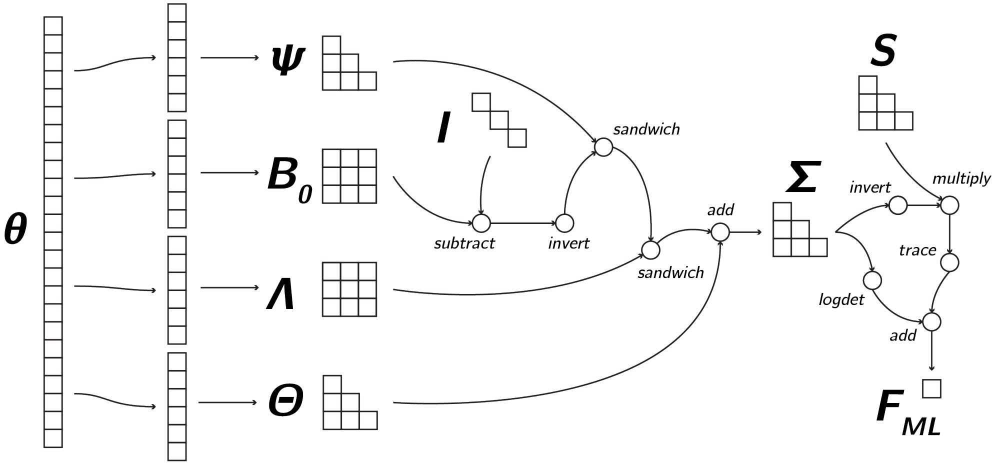
SEM computation graph



SEM computation graph



SEM computation graph

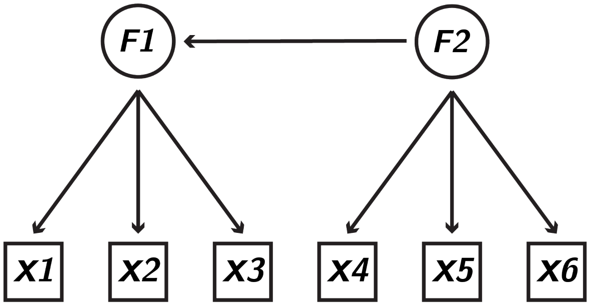


SEM computation graph

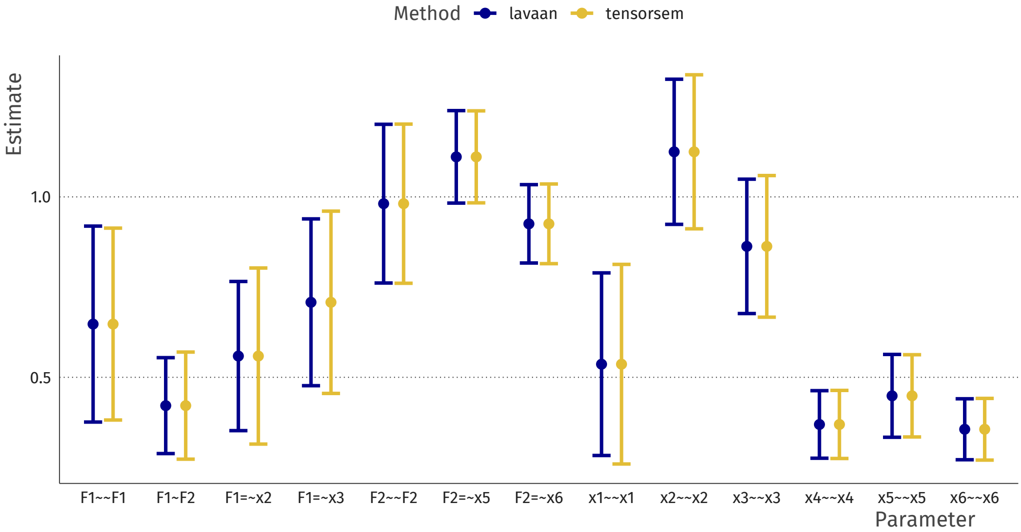
What can we do now

- Get gradient $\nabla F_{ML}(\boldsymbol{\theta})$
- Get Hessian $H_\theta = H\left[F_{ML}(\boldsymbol{\theta})\right]$
- Get standard errors: $SE_\theta \approx \sqrt{diag\left[H_\theta^{-1}\right]}$
- Fit SEM models using smart optimiser (e.g., Adam)

Example



Example



Method
lavaan
tensorsem
Estimate
1.0
0.5
F1~~F1
F1~F2
F1=~x2
F1=~x3
F2~~F2
F2=~x5
F2=~x6
x1~~x1
x2~~x2
x3~~x3
x4~~x4
x5~~x5
x6~~x6
Parameter

Extending SEM

Extending SEM

Now we can edit the objective:

- Different objective
- Penalise structural paths
- Penalise factor loadings

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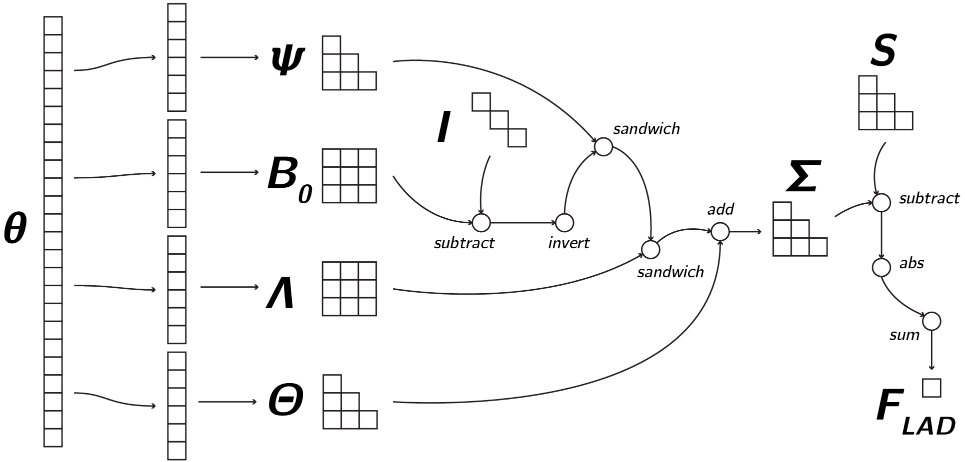
Least Absolute Deviation Estimation in Structural Equation Modeling

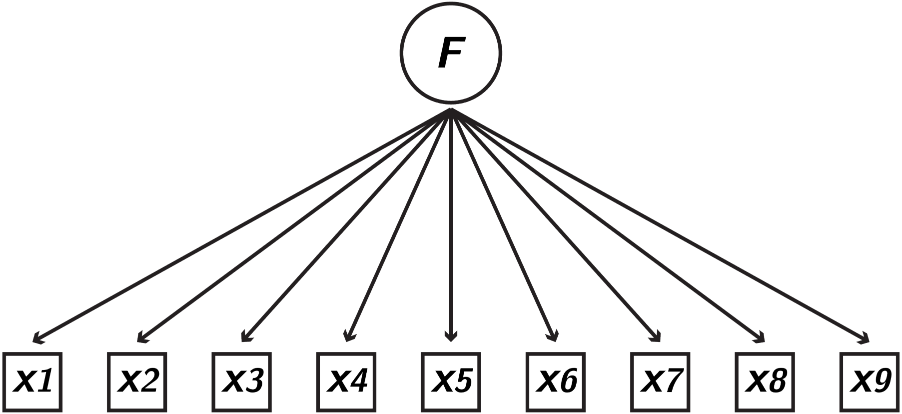
Enno Siemsen
University of Illinois, Urbana-Champaign
Kenneth A. Bollen
University of North Carolina, Chapel Hill

Least absolute deviation (LAD) is a well-known criterion to fit statistical models, but little is known about LAD estimation in structural equation modeling (SEM). To address this gap, the authors use the LAD criterion in SEM by minimizing the sum of the absolute deviations between the observed and the model implied covariance matrices. Using Monte Carlo

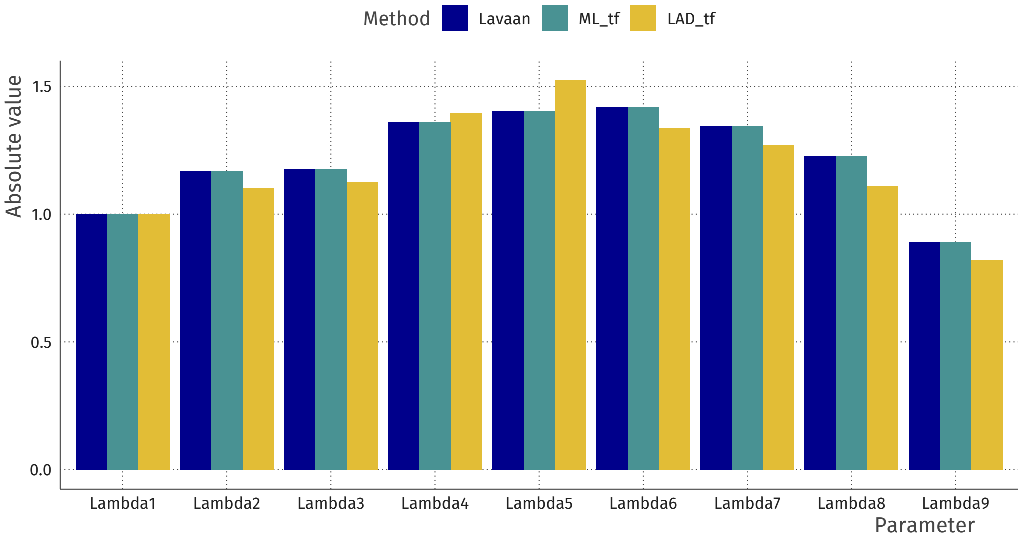
Least absolute deviation estimation



Least absolute deviation estimation



Least absolute deviation estimation



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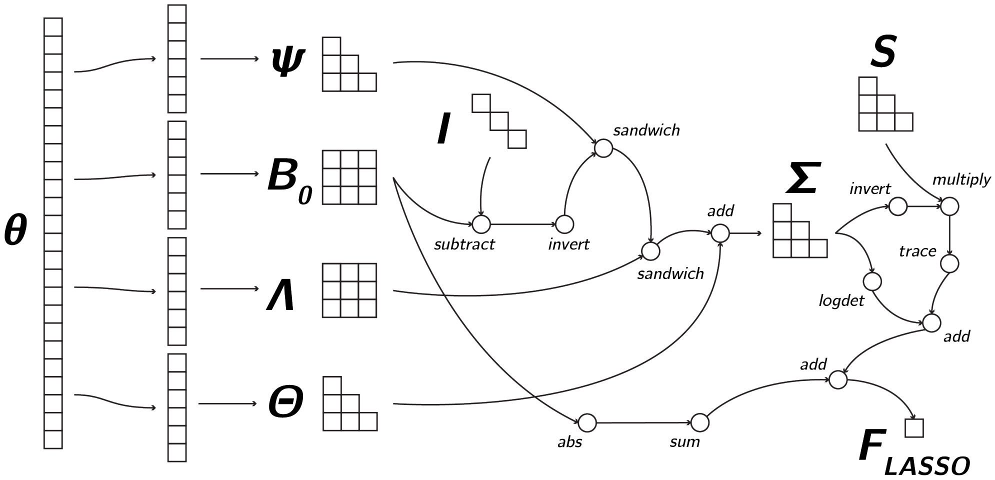
ISSN: 1070-5511 print / 1532-8007 online
DOI: 10.1080/10705511.2016.1154793

Routledge
Taylor & Francis Group

Regularized Structural Equation Modeling

Ross Jacobucci,[1] Kevin J. Grimm,[2] and John J. McArdle[1]
[1]*University of Southern California*
[2]*Arizona State University*

A new method is proposed that extends the use of regularization in both lasso and ridge regression to structural equation models. The method is termed regularized structural equation modeling (RegSEM). RegSEM penalizes specific parameters in structural equation models, with the goal of creating easier to understand and simpler models. Although regularization has gained wide adoption in regression, very little has transferred to models with latent variables. By adding penalties to specific parameters in a structural equation model, researchers have a high level of flexibility in reducing model complexity, overcoming poor fitting models, and the creation of models that are more likely to generalize to new samples. The proposed method was evaluated through a simulation study, two illustrative examples involving a measurement model, and one empirical example involving the structural part of the model to demonstrate RegSEM's utility.



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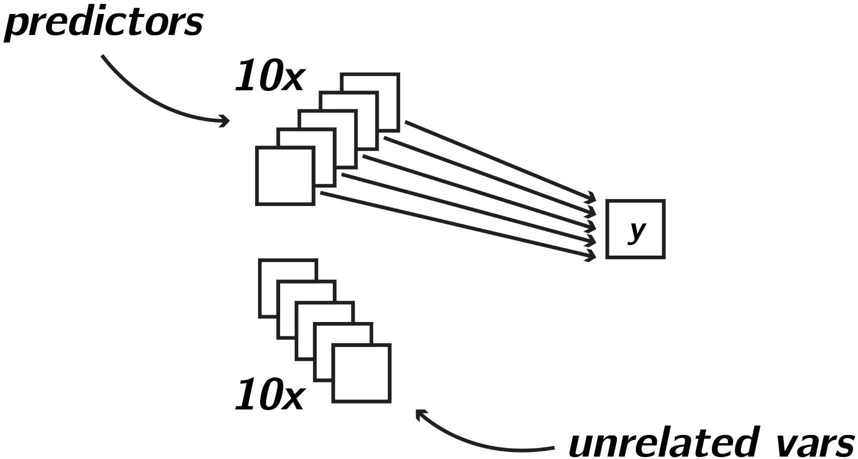
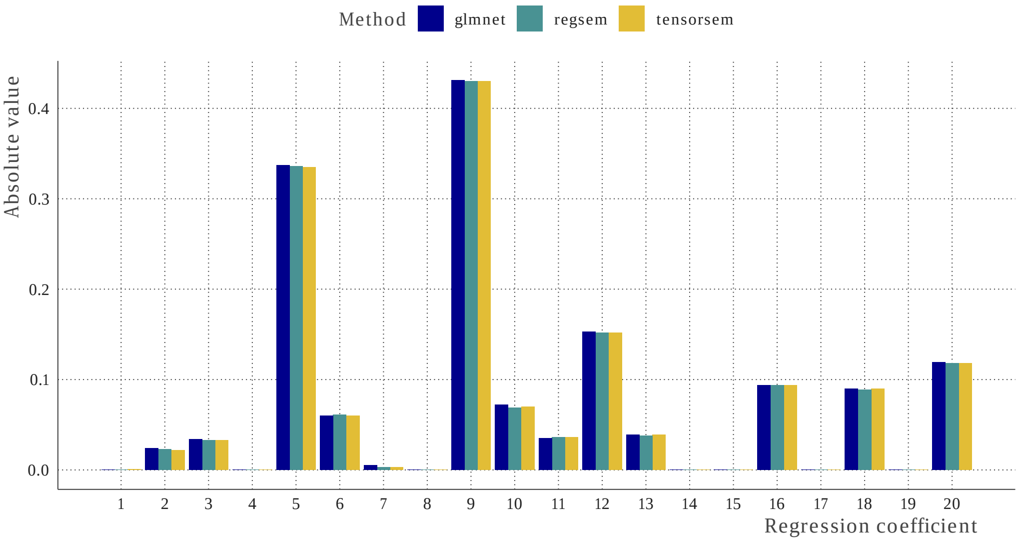
Regularized regression



Regularized regression



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SEPTEMBER 2018
https://doi.org/10.1007/s11336-018-9623-z

APPROXIMATED PENALIZED MAXIMUM LIKELIHOOD FOR EXPLORATORY FACTOR ANALYSIS: AN ORTHOGONAL CASE

SHAOBO JIN

UPPSALA UNIVERSITY

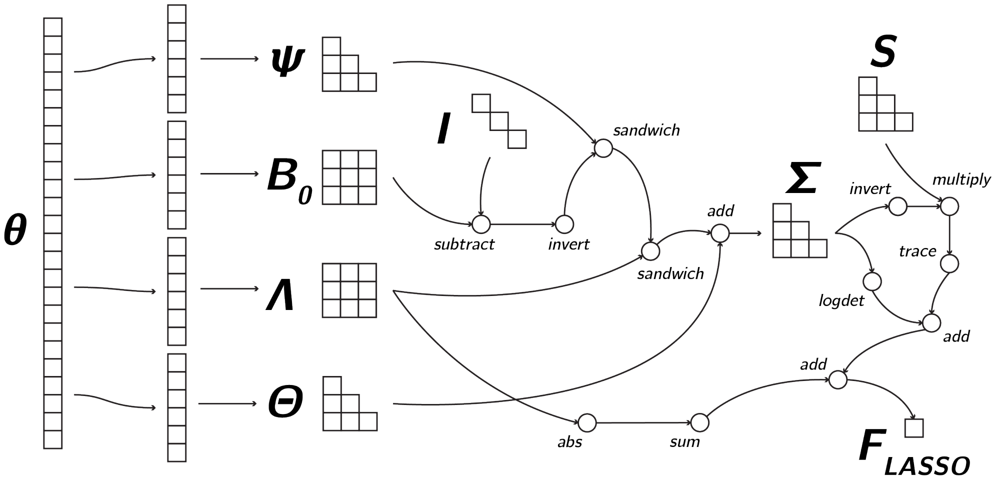
IRINI MOUSTAKI

LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

FAN YANG- WALLENTIN

UPPSALA UNIVERSITY

The problem of penalized maximum likelihood (PML) for an exploratory factor analysis (EFA) model is studied in this paper. An EFA model is typically estimated using maximum likelihood and then the estimated loading matrix is rotated to obtain a sparse representation. Penalized maximum likelihood simulta-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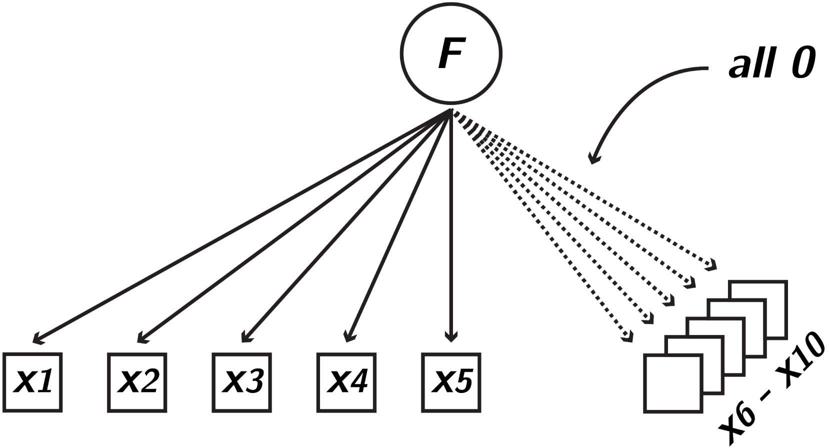
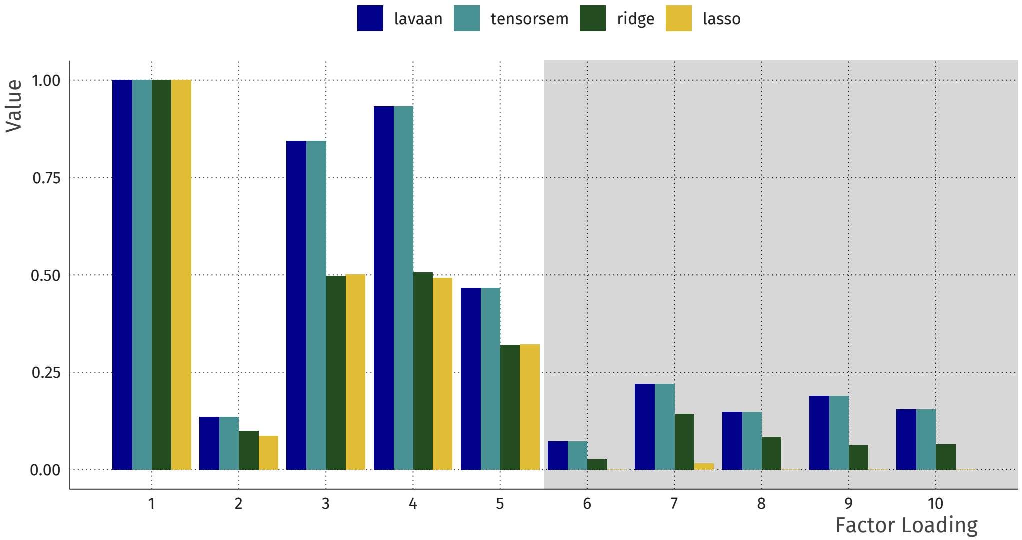
Regularized exploratory factor analysis



Regularized exploratory factor analysis



R package showcase

tensorsem

R package showcase

```r
# Install tensorsem
devtools::install_github("vankesteren/tensorsem@experimental")
library(tensorsem)
```

R package showcase

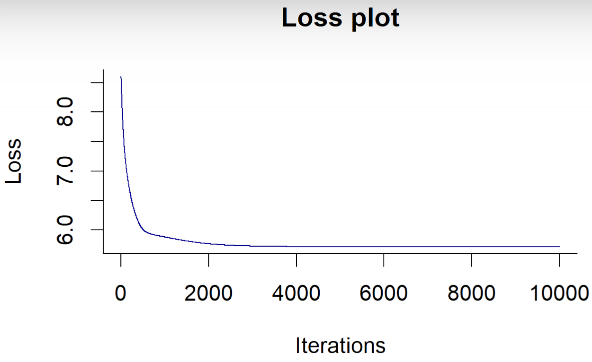
```r
# Create a model using lavaan syntax
mod <- "
F1 =~ x1 + x2 + x3
F2 =~ x4 + x5 + x6
F1 ~ F2
"
dat <- lavaan::HolzingerSwineford1939
```

R package showcase

```
# create a tf_sem object, similar to lavaan
tensem ← tf_sem(lav_model = mod, data = dat)

# optimise / compute parameter estimates
tensem$train(niter = 10000)


> [loss: 5.71860] [==========================>--------] 65%
```

R package showcase

```
# plot the loss over iterations
tensem$plot_loss()
```



R package showcase

```
tensem$summary()


TensorFlow SEM session
----------------------


Loss: 5.718596


Sigma:
          [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
[1,] 1.3583698 0.4594367 0.5818981 0.4136796 0.4595856 0.3828115
[2,] 0.4594367 1.3817839 0.3252543 0.2312279 0.2568872 0.2139740
[3,] 0.5818981 0.3252543 1.2748649 0.2928609 0.3253597 0.2710081
[4,] 0.4136796 0.2312279 0.2928609 1.3506645 1.0901316 0.9080243
[5,] 0.4595856 0.2568872 0.3253597 1.0901316 1.6597858 1.0087876
[6,] 0.3828115 0.2139740 0.2710081 0.9080243 1.0087876 1.1963584


Psi:
          [,1]      [,2]
[1,] 0.6475559 0.0000000
[2,] 0.0000000 0.9812432
```

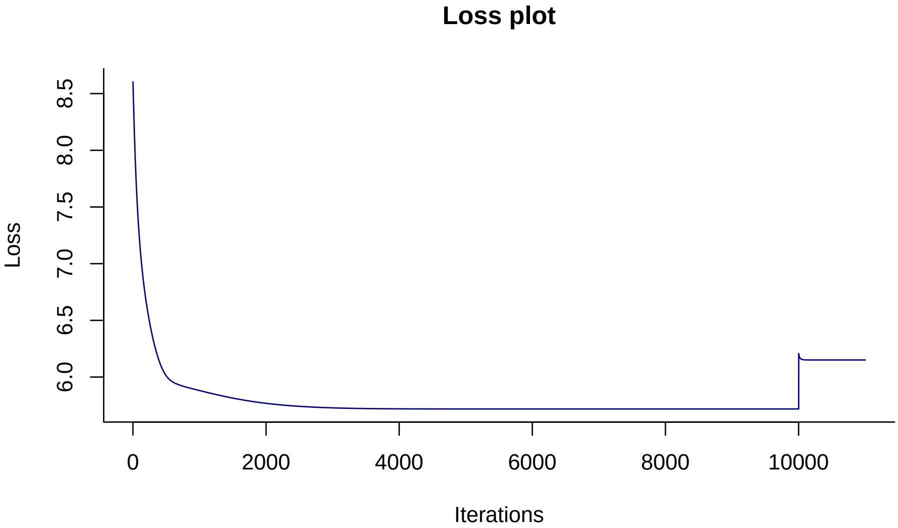
R package showcase

```
Beta:
     [,1]      [,2]
[1,]    0 0.4215872
[2,]    0 0.0000000


Lambda:
          [,1]      [,2]
[1,] 1.0000000 0.0000000
[2,] 0.5589541 0.0000000
[3,] 0.7079415 0.0000000
[4,] 0.0000000 1.0000000
[5,] 0.0000000 1.1109698
[6,] 0.0000000 0.9253816


Theta:
          [,1]    [,2]      [,3]      [,4]      [,5]      [,6]
[1,] 0.5364119 0.00000 0.0000000 0.0000000 0.0000000 0.0000000
[2,] 0.0000000 1.12498 0.0000000 0.0000000 0.0000000 0.0000000
[3,] 0.0000000 0.00000 0.8629151 0.0000000 0.0000000 0.0000000
[4,] 0.0000000 0.00000 0.0000000 0.3694213 0.0000000 0.0000000
[5,] 0.0000000 0.00000 0.0000000 0.0000000 0.4486825 0.0000000
[6,] 0.0000000 0.00000 0.0000000 0.0000000 0.0000000 0.3560894
```

R package showcase

```r
# add ridge penalty to lambda and refit
tensem$penalties$ridge_lambda <- 0.1
tensem$train(1000)
tensem$plot_loss()
```

R package showcase



Future developments

- Stochastic gradient descent
- Batch processing
- Full-information methods / missing data
- Arbitrary penalties on parameters
- Other objective functions
- Dropout regularisation
- Early stopping
- ... (insert your idea)

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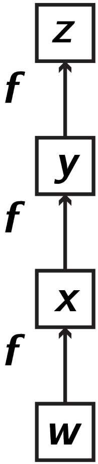
e.vankesteren1@uu.nl
github.com/vankesteren/tensorsem
@ejvankesteren

Automatic gradients

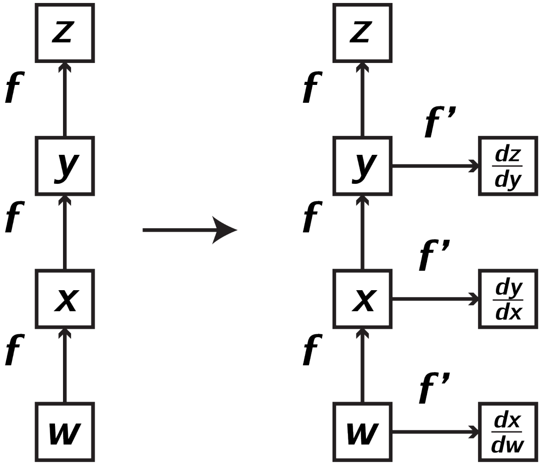
Computation Graphs: gradient computation

Autograd: use the chain rule to traverse the graph from objective back to parameters
Deep learning book section 6.5.1, figure 6.10

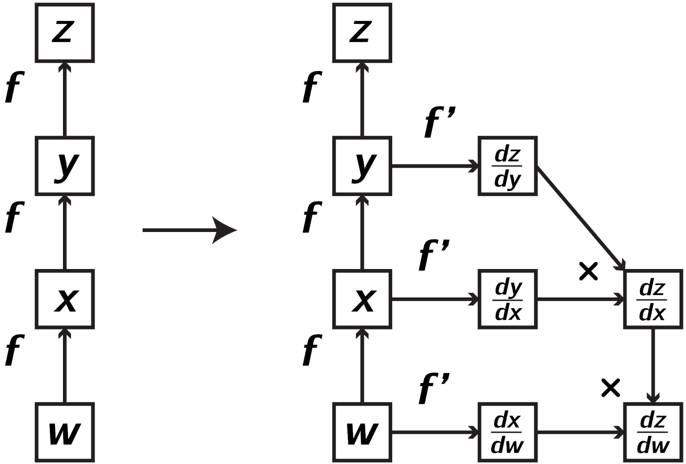
Gradient



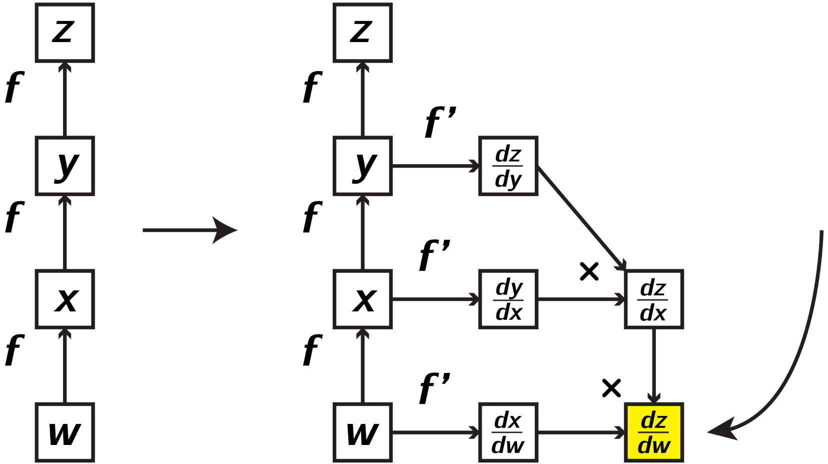
Gradient



Gradient



Gradient



Parameter path for LASSO regression. (Early stopping showcase)